hello

math 152.01

TR 5:30-7:48 • CC 230

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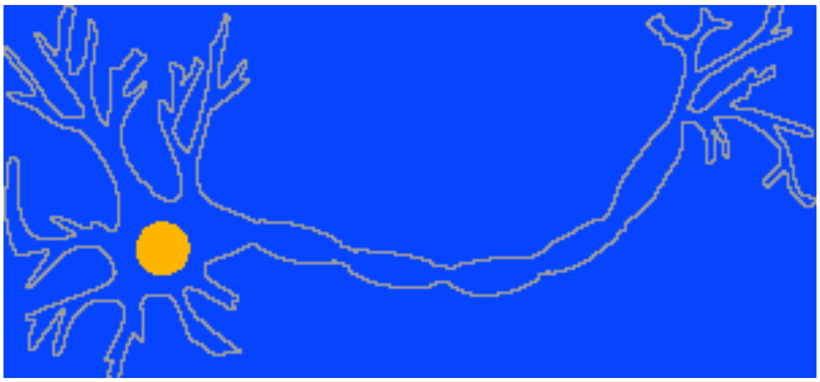
Go over syllabus.
Talk about MSLC,
suggested problems,
show moodle.

why study calculus?

because we live in a changing world



differentiation

- definition
- polynomials
- exponentials
- trigonometric and hyperbolic trig
- sums, differences, products, and quotients
- chain rule

take the derivative to find rates of change

Do a few.

what if I know how things are changing?

example i:

Suppose I drive for one hour at 100 kph.
How far have I traveled?

example ii:

emphasize we can only estimate.

compute with both left-limits and right-limits

draw picture of velocity, connect to area under curve

observe x-axis is time, y-axis is distance/time, so area is in distance

Every ten seconds I wrote down my speed in meters per second. How far did I drive in the first ninety seconds? Be careful.

t	0	10	20	30	40	50	60	70	80	90
v	5	25	15	10	10	15	20	20	25	10

example iii:

What is the area bounded by the curve

$$y = x^2$$

the x-axis and the line $x = 6$?

Do with n=1, n=2, n=3, n=6, write for general n (without sigma notation) (Use right end points for n)

Ask if left/right is under/over estimation.

Introduce notation as a way to simplify sums.

Do a few examples.

Σ - notation

Prove first two on left, comment about all on right.

Use these rules to evaluate some sums.

For more practice, see Appendix E.

formulae

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=1}^{n} c = n\,c$$

$$\sum_{i=1}^{n} c\,a_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

example iii (continued):

What is the area bounded by the curve

$$y = x^2$$

the x-axis and the line $x = 6$?

Write the formula for n in sigma notation, then simplify, then take the limit.

(Correct answer=6^3/3)

This act of computing the area is such an important one, that we give it its own name: integration.

definition

If f is a continuous function defined for $a \le x \le b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0 = a$, x_1, x_2, $\ldots x_n = b$ be the endpoints of these subintervals and we choose sample points x_1^*, x_2^*, $\ldots x_n^*$ in these subintervals so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral** of f from a to b is

$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Point out that this is exactly what we just did (where x_i* were either left or right end points).

Note that if f is positive, this is the area under the curve (so write the x^2 result in this notation), so we can be clever... integrate, say, sqrt(9-x^2) from x=0 to 3.

Comment that since f can be negative, we're computing signed area rather than area.

Say that these sums are called Riemann sums.

properties

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b (f(x)-g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Point out that these follow immediately from the properties of sums... integration is kind of like adding infinitely many infinitely small pieces.

Introduce the Midpoint rule.

Evaluate a few integrals.

properties

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$.

If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

First two are obvious from the picture...

Third follows by h=g-f

Fourth from third by g=m, g=M

Use fourth to estimate integral of sqrt(x) from 4 to 9.

next time

- quiz i: algebra and differentiation review
- attempt webwork 0, start webwork 1
- read §§ 5.1 - 5.4
- try suggested problems for 5.1 and 5.2
- we will discuss the fundamental theorem of calculus and introduce indefinite integrals